Complex numbers: A complex no is one having real and imaginary parts e.g. $a + ib \; where \; a, b \; E \; R \; and \; i \; E \; imaginary \; i$ defined by the statement $i^2 = -1$.

This is obtained by the solution of the quadratic equation $x^2 + 1 = 0$

Recall: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For real roots, $b^2 - 4ac > 0$

For equal roots; $b^2 - 4ac = 0$

For complex roots; $b^2 - 4ac < 0$

$x^2 + 1 = x^2 + 0x + 1 = 0$

$$x = \frac{-0 \pm \sqrt{0^2 - 4}}{2} = \frac{\pm\sqrt{-4}}{2}$$

$$= \frac{\pm\sqrt{4i^2}}{2}$$

$$x = \frac{\pm 2i}{2} = x = - \pm i$$

Ex: (1) $x^2 - 4x + 13 = 0$

$x^2 + 2x + 5 = 0$

$x^2 + x + 1 = 0$

$x^2 \pm 4 = 0$

$x^2 \pm 9 = 0$

$x^2 \pm 25 = 0$

Operations on complex numbers

Addition of 2 complex numbers

$(2 + 4i) + 3i = 2 + 7i$

$(3 + 10l\, i) + (4 - 24i) = 7 + 77i$

$(1 + i) + (-3 - 4i) = -2 - 3i$

$Z_1 = a_1 + ib$

$Z_2 = a_2 + ib_2$

$Z_1 + Z_2 = (a_1 + a_2) + i\,(b_1 + b_2)$

When you are adding in case of $2 ¢\, X\, nos$

$Z_1 - Z_2 = (a_1 - a_2) + i(b_1 + b_2)$

$if\ Z_1 = 10 + 7i\ and\ Z_2 = 7 + 3i$

$\therefore Z_1 - Z_2 = 3 + 4i$

$Z_2 - Z_1 = -3 - 4i$

$Z_1 - Z_2 \neq Z_2 - Z_1\ \therefore it\ does\ not\ obey\ commutativity\ law$

$The\ product\ of\ 2 ¢\, X\, nos$

$Z_1.Z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$= (a_1a_2 - b_1b_2) + i(a_2b_1 - a_1b_2)$

e.g. $if\ Z_1 = 3 + 3i, multiply\ Z_1 by\ its\ conjugate\ (3 + 3i)(3 - 3i)$

Definition: The conjugate of the $\not\subset X$ $no\ a + ib\ is\ a - ib$

When you multiply a complex no with its conjugate you get a real no

e.g. $(a_1^2 - a_1 i b_1 + a_2 i b_1 - i^2 b_1^2$

$a_1^2 + b_1^2$

Division of 2 $\not\subset X\ no$

$$\frac{Z_1}{Z_2} = \frac{a_1 + ib_1}{a_2 + ib_2}\ iff\ Z_2 \neq 0$$

i.e. rationalize the denominator

$$= \left(\frac{a_1 + ib_1}{a_2 + ib_2}\right)\left(\frac{a_2 - ib_2}{a_2 - ib_2}\right)$$

$$= \frac{a_1 a_2 - i a_1 b_2 + i b_1 a_2 - i^2 b_1 b_2}{{a_2}^2 - i a_2 b_2 + i b_1 a_2 - i^2 {b_2}^2}$$

$$= \frac{a_1 a_2 + b_1 b_2 + i(b_1 a_2 - a_1 b_2)}{{a_2}^2 + {b_2}^2}$$

$$= \frac{a_1 a_2 + b_1 b_2}{{a_2}^2 + {b_2}^2} + i\left(\frac{b_1 a_2 - a_1 b_2}{{a_2}^2 + {b_2}^2}\right)$$

Or

$$= \frac{a_1 a_2 + b_1 b_2}{{a_2}^2 + {b_2}^2} - 1\left(\frac{a_1 b_2 - b_1 a_2}{{a_2}^2 + {b_2}^2}\right)$$

$* \ Ex: Simplify\ the\ following:$

$$\frac{2 + i}{1 + i}, \frac{5 - 2i}{-1 + 1}, \frac{a + bi}{c + di}$$

Complex conjugate

The $\not{c}X\ nos\ a+ib\ and\ a-ib\ are\ known\ as\ conjugate\ \not{c}X\ nos\ and\ their$

$product$ is the real no $a^2 + b^2$

Symbol used to denote conjugate $\not{c}X$ no Z is $\bar{Z}$. If Z = a + ib, $\bar{Z}$ = a – ib

Equality of $\not{c}X\ nos$

To prove that if 2 $\not{c}X\ nos\ a+ib\ and\ c+id\ are\ equal\ then\ a=c\ and\ b=d$

a + ib = c + id

(a – c) + ib – id = 0

(a – c) + i(b – d) =m 0

Or

On squaring (a line has been omitted)

$(a-c)^2 + (d-b)^2 = 0$

Analogy: If $x\ and\ y$ are real and $x^2 + y^2 = 0\ , then\ x = 0\ and\ y = 0$

Using the above, we have

$a - c = 0\ and\ d - b = 0$

$\Longrightarrow\gg\ \ a = c\ amd\ d = b$

Hence if 2 $\not{c}X\ nos\ are\ equal\ their\ real\ part$ and imaginary part are equal simultaneously.

$*\ Ex$: Express in the form $a + ib$ where a and b are both real

1) $(3+2i)(7-5i)$

2) $\frac{i-2}{2-3i}\ \frac{-2+i}{2-3i}$

3) $\frac{2+i^2}{2-i}$

4) $\left(\frac{1-2i}{(4-3i)^2}\right)$

5) $\frac{L+2i}{i^3(1-3i)}$

(Cos 150^0 + i Sin 150^0) (Cos 60^0 + i Sin 60^0)

6) Show that = Cos 210^0 + i Sin 210^0 = $-\frac{\sqrt{3}}{2} - i\frac{1}{2}$

* Solve: (1) $x^2 + 3x + 10 = 0$

(2) $x^2 + 4x + 8 = 0$

(3) $x^2 \pm x + 1 = 0$

(4) $x^2 + 1 = 0$

(5) Show that: (a) $i^7 = i, (b)\ i^5 = i \quad (c)\ i^9 + 2i = -i^{13}$

(6) Show that $x^3 - 1 = 0$ *has* 3 *solution*

Viz 1, -$\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

(7) Show that the quadratic equation $x^4 - 1 = 0$

Has 4 solutions viz: $\pm$, i *and* $-i[(x^4 - 1) \equiv (x^2-1)(x^2+1)]$

(8) *Show that* $\frac{2+3i}{4+5i} = \frac{1}{41}(23+2i)$

Square roots of negative Nos

Definition: $\sqrt{-1} = \sqrt{i^2} = i$

$\sqrt{-9} = \sqrt{9i^2} = 3i$

Algebra of $₵X$ nos

The fundamental rules of algebra used in the manipulate of real nos are

1. The commutative law of addition

   a + b = b + a

2. The associative law of + (addition)

   (a + b) + c = a + (b + c)

3. The associative law of multiplication

   (ab) c = a(bc)

4. The distributive law of x (multiplication)

   (a +b) c = ac + bc)

* (A) Ex: (1) is addition and subtraction commute in the set of $₵X$ nos?

  (2) is $\pm$ associative in the set of $₵X$ nos?

  (3) What is the inverse of $4 + 4i$?

(B) 1. Does the operation + on $₵X$ have an identity element? If so, name it.

  (2) For each element in a set of $₵X$ $nos,$ is there an inverse

Let Z = $a + ib, \ a, \ b \in R$

Then Z = 0 $\Rightarrow$ $Z\bar{Z} = 0$

$\Rightarrow$ $a^2 + b^2 = 0$

$\Rightarrow$ a = 0, b = 0

Then $Z_1 = Z \Rightarrow Z_1 - Z = 0$

$\Rightarrow (a - a_1) + i\,(b - b,) = 0$

$\Rightarrow a = a,; b, = b$

$Z + Z_1 = (a + a,) + i\,(b + b,)$

$Z\,Z_1 = (a, a - b\,b,) + i\,(ab, -a, b)$

Note: $i^2 = -1$

$Z\ can\ be\ regarded$ as the ordered number

-pair (a, b) ordered because $(x, y) \neq (y, x)$

i.e. $(3, 5) \neq (5,3)$

or $a + ib \neq b + ia$

Now a $\not\subset X\ no$ is defined as an ordered pair of real nos and is represented by the symbol (a, b)

Rules for Operation

i. (a, b) = (x, y) only $a = x\ and\ b = y$

ii. (a, b) + $(x, y) = (a + x, b + y)$

iii. (a, b) x $(x, y) = [(ax - by), (ay + bx)]$

$[x, o]$ *is called a real* $\not{c}X$ *no*$: x$

$[0, y]$*is called a pure imaginary* $\not{c}X$ *no*$: y$

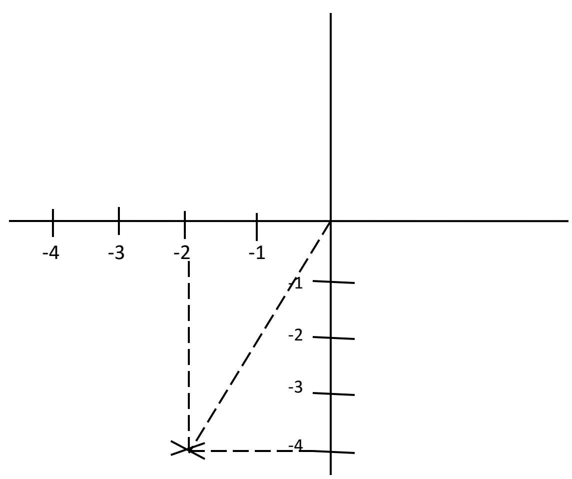
$* \ Ex$: *Compute the ff*$: [0,1]X\ [0,1]$

$[0,1]\ X\ \ [1,0]$

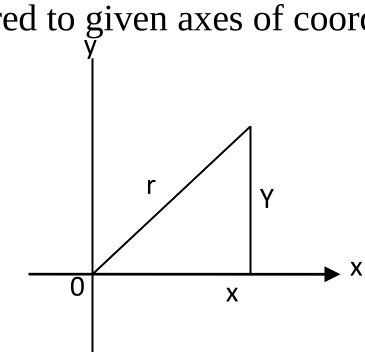
$[1,0]X\ [0,1]$*and* $[1,0]X\ [1,0]$

THE ARGAND DIAGRAM

Is a graphical representation of complex no e.g.: $-2-4i$



The $\not{c}X$ *no* $Z = a + ib$ is an ordered number –pair [a, b] and it can be represented by the pt (a, b) or (x, y) referred to given axes of coordinate



The pt $Z\ (x, y) or\ (a, b)$ represents the $\not{c}X\ no\ Z,$ and there is a one-to-one correspondence below the $\not{c}X$ nos [Z] and the points [Z] of the Cartesian plane.

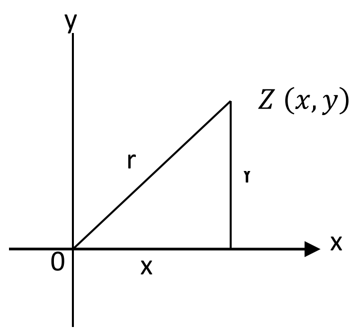
The geometrical representation of $\not{c}X$ consisting of [Z] onto the plane is called the Argand diagram.

{J.R. Argand 1768-1822}

$\not{c}X$ nos are mapped in 2 directions while vectors are 3 dimentional

modulus or absolute value

suppose $Z = (x, y)$represents the $\not{c}X$ nos $Z =$. Let r be the real no given by r = $\sqrt{x^2 + y^2}$, so that r is +ve and is equal to the length OZ



The no r is called the modulus of Z and is written $/Z/$

$r = \sqrt{x^2 + y^2} = /Z/$

* Take note: The modulus of any no is a positive

e.g. The modulus of 3 + 4i = 5

and $3 - 4i = 5$

Let $x\,\hat{O}Z$ be measured positively in the anticlockwise direction and suppose θ is the real no, modulus 2π, such that $\theta = X\,\hat{O}Z$

Then $\hat{O}$ is called the argument of Z and is written arg. $\hat{Z}$. The value of θ is measured or determined by the two equations.

Cos $\theta = \frac{x}{r}, Sin\ \theta = \frac{y}{r}$

Now, $Z = x + iy$

$= r\ (Cos\ \theta + i\ Sin\ \theta) -$ Euler representation of ¢X nos

$= (r, \theta) -$ polar form

The three forms of ¢X nos

1. The rectangular form $Z = x + iy$
2. The polar form $Z = r\ (Cos\ \theta + i\ Sin\ \theta) = r\ Cis\ \theta$
3. The exponential form: $r = e^{i\theta}$

* Represents the following ¢X no on the Argand diagram and express them in polar form.

i. $Z_1 = 4 + 31$

ii. $Z_2 = 2$

iii. $Z_3 = 1 - 3i$

iv. $Z_4 = -2 + 2i$

v. $Z_1 + Z_2,\ Z_2 + Z_3 + Z_2,\ Z_1 + Z_4 - Z_3$

The polar form

Let $Z = x + iy$, the polar equivalent is $Z = r\ (Cos\ \theta + i\ Sin\ \theta)$

Where r is called the modulus or amplitude or the length of Z, written $/Z/$ or mod Z.

Where r = 1, Z lies on the unit circle in the number plane with centre at the origin.

θ is called argument and is defined as $\theta\ tan^{-1}\frac{y}{x}$

e.g. Z = 4 + 3i

$= 5\ (Cos\ \theta + i\ sin\ \theta)$

$$\theta = tan^{-1}\frac{3}{4}$$

$= 36..86$

$Z = 5\ Cis$ (37) approx

$Arg\ Z = 37^0$

The Argand diagram would be needed to determine the principal value.

If $Z = 3 + 4i$, then the location is such that Z is in the first quadrant

$$0 \leq Arg\ Z \leq \frac{\pi}{2}$$

i.e. it lies between 0 and 90^0

∴ The principal value of arg Z is 37^0

Arg Z = 37^0

$/Z/ = 5$

$Z = 5\,(Cos\,37^0 + i\,Sin\,37^0)$

$= 5\,Cis\,37^0$

General value of θ is $37 \pm \pi r$

To get the absolute value, we sketch the Argand diagram

* Ex: Find the modulus and principal value of: (a) $Z = \frac{3+4i}{3-4i}$, hence express Z in polar form.

(2) $Z = 3 + 4i$

(3) $Z = 3 - 4i$

Solution:

1. $/Z/ = 1, 106^0$
2. $/Z/ = 5,\ 53^0$
3. $/Z/ = 5, 324^0$

Operations with polar form is operations of $\not{c}X$ nos in the Eulerian representation

Addition: Let $Z_1 = x_1 + iy_1$

$= Z_2 = x_2 + iy_2$

$Z_1 + Z_2 = (x_1 + x_2) + i(y_1 + y_2)$

If $Z_1 = r_1\, Cis\, \theta_1\, and\, Z_2 = r_2\, Cis\, \theta_2$

$Then\, (Z_1 + Z_2) = r_1\, Cis\, \theta_1 + r_2\, Cis\, \theta_2$

$= r_1\, (Cos\, \theta_1 + i\, Sin\, \theta_1) + r_1\, (Sin\, \theta_1 + r_2\, Sin\, \theta_2)$

$Subtraction: Z_1 - Z_2\ (x_1 - x_2) + i(y_1 - y_2) \neq (Z_2 + Z_1)$

$Also, (Z_1 - Z_2) = r_1\, Cis\, \theta_1 - r_2\, Cis\, \theta_2$

$= (r_1\, Cos\, \theta_1 - r_2\, Cos\, \theta_2) + i(r_1\, Sin\, \theta_1 - r_2\, Sin\, \theta_2)$

$Multiplication$:

$Z_1.Z_2 = (r_1\, Cis\, \theta_1).(r_2\, Cis\, \theta_2)$

$= r_1 r_2\, [(Cos\, \theta_1 + i\, Sin\, \theta_1)(Cos\, \theta_2 + i\, Sin\, \theta_2)]$

$= r_1 r_2 [Cos\, (\theta_1 + \theta_2) + i\, Sin\, (\theta_1 + \theta_2)]$

$= r_1 r_2\, \{Cis\, (\theta_1 + \theta_2)\}$

Note : Knowledge of Trig. Is referred

* Thus if we are multiplying 2 $\not{c}X$ nos in polar moduli and the sum of their arguments.

1. e.g. if $Z_1 = 2\, Cis\, 60^0\, and\, Z_2 = 3\, Cis\, 45^0$,

Then $Z_1.Z_2 = (2\, Cis\, 60^0) X\, (3\, Cis\, 45^0)$

$= 6\, Cis\, 105^0$

$= 6(Cos\, 105^0 + i\, Sin\, 105^0)$

$= 6(-Cos\, 75^0 + i\, Sin\, 75^0)$

$= 6\, (-0.259 + i\, 0.966$

$= -1.554 + i\ 5.796$

2. (2 Cis 30^0) (3 Cis 60^0) = 0 + 6 i

3. (4 Cis 20^0) (6 Cis 40^0) = 12(1 + $i\sqrt{3}$

4. $Z.Z = (r\ Cis\ \theta)^2$

$= (r\ Cis\ \theta)^2 = Z^2$

$= (r\ Cis\ \theta)\ (r\ Cis\ \theta)$

$= r^2 Cis\ (2\theta)$

$Z^2.Z = r^3 Cis\ 3\theta$

$Z^n = r^n\ Cis\ n\theta$ $\quad\quad\quad$ $(Demoivre's\ theorem)$

In particular

$Z^n = [r(Cos\ \theta + i\ Sin\ \theta)]^n = r^n\ (Cos\ \theta + i\ Sin\ \theta)^n$

$Z^n = r^n\ (Cos\ n\theta + i\ Sin\ n\theta)$

Thus, in raising a $\not{c}X$ no Z to the power n, the absolute value, r, of the no is raised to the power n and the argument θ of Z is multiplied by n.

e.g. $Z = 1 + i = \sqrt{2}\ (Cos\ 45^0 + Sin\ 45^0)$

$Z^2 = 2\ (Cos\ 90^0 + i\ Sin\ 90^0) = 2i$

$Z^3 = -2 + 2i$

Division of $\not{c}X$ nos

$Z_1 \div Z_2 = \frac{Z_1}{Z_2}$

Let $Z_1 = r_1 \, Cis \, \theta_1 = r_1 \, (Cos \, \theta_1 + i \, Sin \, \theta_1)$

$and \; Z_2 = r_2 \, Cis \, \theta_2 = r_2 \, (Cos \, \theta_2 + i \, Sin \, \theta_2)$

$$\frac{Z_1}{Z_2} = \frac{r_1 \, Cis \, \theta_1}{r_2 \, Cis \, \theta_2} \qquad (rationalize)$$

$$= \left(\frac{r_1}{r_2}\right)\left(\frac{Cis \, \theta_1}{Cis \, \theta_2}\right).\frac{-Cis \, \theta_2}{-Cis \, \theta_2}$$

$$= \frac{r_1}{r_2}\left[\frac{(Cos \, \theta_1 + i \, Sin \, \theta_1)(Cos \, \theta_2 - i \, Sin \, \theta_2)}{(Cos \, \theta_2 + i \, Sin \, \theta_2)(Cos \, \theta_2 - i \, Sin \, \theta_2)}\right]$$

$$= \frac{r_1}{r_2} \, Cis \, (\theta_1 - \theta_2)$$

$$\frac{r_1}{r_2}\left[\frac{Cos \, \theta_1 \, Cos \, \theta_2 + Sin \, \theta_1 \, Sin \, \theta_2 - i(Cos\theta_1 Sin\theta_2 - Cos\theta_2 Sin\theta_1}{Cos^2\theta_2 + Sin^2\theta_2}\right]$$

$$\frac{r_1}{r_2}\left[\frac{Cos \, \theta_1 \, Cos \, \theta_2 + Sin \, \theta_1 \, Sin \, \theta_2 - i(Sin\theta_1 Cos\theta_2 - Cos\theta_1 Sin\theta_2}{Cos^2\theta_2 + Sin^2\theta_2}\right]$$

$$= \frac{r_1}{r_2} \, [Cos(\theta_1 - \theta_2) - i \, Sin \, (\theta_1 - \theta_2)]$$

$$= \frac{r_1}{r_2} \, [Cis \, (\theta_1 - \theta_2)]$$

Example: $\frac{5Cis\left(\frac{\pi}{4}\right)}{3Cis\left(\frac{\pi}{6}\right)} = \gg \frac{5Cis\left(\frac{\pi}{4}\right)}{3Cis\left(\frac{\pi}{6}\right)} = \frac{5}{3} Cis \left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{5}{3} Cis \frac{\pi}{2}$

$$= \frac{5}{3} \, Cis \, 15^0 = \frac{5}{3}(Cos \, 15^0 + i \, Sin \, 15^0)$$

Thus in dividing a $\not{c}X\ no\ Z_1$ by a $\not{c}X\ no\ Z_2$ the absolute value $r_1\ of\ Z_1$ is divided by absolute value $r_2\ of\ Z_2$ and the argument θ_2 is subtracted from the argument θ_1 of Z_1

* (A) if $Z_1 = r_1 (\text{Cos}\ \theta_1 + i\ Sin\ \theta_1)$

And $Z_2 = r_2 (\text{Cos}\ \theta_2 + i\ Sin\ \theta_2)$

Find: (i) $Z_1.Z_2$ (2) $Z_1.Z_2$ (3) $Z_2.Z_1$ (4) $\frac{Z_1}{Z_1.Z_2}$ (5) $\frac{Z_2.Z_2}{Z_1}$ (6) $\frac{Z_2}{Z_1}$

(B) If $Z_1 = \frac{3}{4} Cis\ 25^0\ and\ Z_2 = \frac{5}{6}\ Cis\ 125^0$

Find: (1) $Z_1.Z_2$ (2)${Z_1}^2$ (3) $\frac{Z_1}{Z_2}$ (4) $\frac{Z_1\,Z_2}{Z_2 Z_1}$

DE MOVRE'S THEOREM

$\forall$ $(For\ all)$ rational values of n

$[(Cos\ \theta + i\ Sin\ \theta)]^n = r^n\ (Cos\ n\theta + i\ Sin\ n\ \theta)$

i.e. $(r\ Cis\ \theta)^n = r^n\ Cis\ (n\theta)$

Proof (Case 1)

When n is a +ve integer using the product of 2 $\not{c}X$ nos we have

$(r_1 Cis\ \theta_1)\ (r_2 Cis\ \theta_2) = r_1 r_2\ Cis\ (\theta_1 + \theta_2)$

Also,

$$r_1 r_2\ Cis\ (\theta_1 + \theta_2)\ (r_3\ Cis\ \theta_3)\ r_1 r_2\ r_3\ Cis\ \theta_1 + \theta_2 + \theta_3)$$

Proceeding in the same way, we have

$(r_1 Cis\ \theta_1\ )\ (r_2 Cis\ \theta_2)\ (\dots\dots\dots.) = \ r_1 r_2\ \ \dots\dots\dots..Cis\ (\theta_1 + \theta_2 + \cdots \dots)$

But if $r_1 = r_2$ ………..and $\theta_1 = \theta_2 = \cdots \dots \dots ..$

$= (Cis\ \theta)^n = \ r^n\ Cis\ (n\theta)$

Case II

When n is negative no let n = -m, where m is a +ve integer

$r^n\ (Cis\ \theta)^n = (\ r\ Cis\ \theta)^{-m}$

$$= \frac{1}{(r\ Cis\ \theta)^m}$$

$$= \frac{1}{r^m Cis\ (m\theta)}$$

$$\frac{1}{r^m Cis\ (m\theta)}\ \frac{-(Cis\ m\theta)}{-(Cis\ m\theta)}$$

$$\frac{1}{r^m Cis\ m\theta}\ .\frac{(Cis - m\theta)}{(Cis - m\theta)}$$

$$\frac{1}{r^m(Cos\ m\theta + iSin\ m\theta)}.\left(\frac{Cos\ m\theta - i\ Sin\ m\theta}{Cos\ m\theta - i\ Sin\ m\theta}\right)$$

$$= \ \frac{Cos\ m\theta - i\ Sin\ m\theta}{r^m(Cos^2 m\theta + \ Sin^2\ m\theta)}$$

$$= \ \frac{Cos\ m\theta - i\ Sin\ m\theta}{r^m(1)}$$

$$= \frac{1}{r^m}\ (Cos\ m\theta - i\ Sin\ m\theta)$$

$r^n\ (Cos\ m\theta - i\ Sin\ m\theta)$

$r^n\ (Cos\ n\theta + i\ Sin\ n\theta)$

Case III

When n is $\pm$ fraction

Let n = $\left(\frac{p}{q}\right), where\ q$ is a $\pm$ve integer

$$\frac{[r\ \left(Cos\ \left(\frac{Q}{q}\right) + \ i \sin\left(\frac{Q}{q}\right)\right]^p = \ r^D(Cos\ \left(\frac{PQ}{q}\right) + \ i\ Sin\ \left(\frac{PQ}{q}\right)}{\left[r\left(Cos\frac{Q}{q}\right) + \ i\ Sin\left(\frac{Q}{q}\right)\right]^q = \ r^q\ (Cos\ \theta + i\ Sin\ \theta)}$$

$Cos\ \theta + i\ Sin\ \theta$

$iff\quad r = 1$

But

$$\left(Cis\frac{Q}{q}\right) = (Cis\ \theta)^{\frac{1}{q}} \ \Rightarrow\!\!\!> Cos\ \left(\frac{Q}{q}\right) + \ i\ Sin\ \left(\frac{Q}{q}\right)$$

Raise to power P

$$(Cis\ \theta)^{\frac{p}{q}}\ \left[Cos\ \left(\frac{Q}{q}\right) + \ i\ Sin\ \left(\frac{Q}{q}\right)\right]^P$$

$$= Cis\ \left(\frac{PQ}{q}\right)$$

Substituting for n, we have

$$(Cis\ \theta)^n = Cis\ (n\theta), where\ n = \frac{p}{q}$$

$\therefore\ from\ the\ above\ three\ cases$

$(Cis\ Q)^n = Cis\ (nQ)$

∀ integral values of n

Assignment

1(a) Express $\frac{(2-i)(3+i)}{(1+2i)(2-3i)}$ $in\ the\ form\ (A + iB)$

Where A and B are real numbers

(b) Describe the loc i represented by the equation:

$(i)\quad /Z/ -1/= 2$

$(ii)\ /Z + 1/=/Z - 1/$

$where\ Z$

$= x + iy\ is\ a\ point\ in\ the\ Argand\ diagram\ derive\ the\ cartesian\ equation$

$of\ the\ loc\ i$

$(c)\ If\ Z = Cos\ \theta + i\ Sin\ \theta, show\ that$

$$Z^n + \frac{1}{Z^n} = 2\ Cos\ n\theta$$

By expanding $(Z + \frac{1}{Z^n})^4\ show\ that\ 16\ Cos^4\theta = 2\ Cos\ 4\theta + 8Cos\ 2\theta + 6$

$(d) Use\ the\ relation\ to\ evaluate\ \int_0^{\frac{11}{4}} Cos^4\theta\ d\theta$

Assignment

* Use the principle of mathematical induction to proof DeMoivores theorem

Application of DeMoivres

I. It can be used to find $Cos\ n\theta\ and\ Sin\ n\theta\ in\ terms\ of\ Cos\ \theta\ and\ Sin\theta$ only if n be a +ve integer.

$Let\ Z = Cis\ \theta$

II.